THE
MATHEMATICAL GAZETTE.

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ON THE MULTIPLICATION OF DECIMALS.

So-called "standard form."

7	·325 × 8·23
8	·23
58	·600
1	·4650
	·21975
60	·28475

Method I have used with success as regards clearness and accuracy. (I believe this is Mr. Pendlebury's arrangement.)

Rule. Put *singles* (or units) figure of multiplier under the *last* figure of multiplicand.

7	·325
	8·23
58	·600 00
1	·465 00
0	·219 75
60	·284 75

Arguments in favour of this second plan:

Instead of such a mechanical rule as Mr. Borchardt's (p. 69, Borchardt's *Arithmetical Types and Examples*), we get the continued use of the sound and fundamental rule of "putting each first figure of a partial product underneath the figure to which that partial product is due." *And this also corresponds to the method of algebraic multiplication.*

We are obeying the natural law that 5 thousandths multiplied by 8 singles (see example above) produce 40 thousandths, *i.e.* 4 hundredths; a '0' is placed where the thousandths column originally (*i.e.* in multiplicand) stood, and 4 hundredths carried on to the hundredths column.

Further—whatever the multiplier (*e.g.* 0·0423), no change in the form of it is necessary: thus

```
·7307 × ·0423
     0·0423
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·02922800
·00146140
·00021921
-----------
·03090861
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whereas (Borchardt, page 69, as *supra*) requires ·0423 to be altered to 4·23, and ·7307 to be turned into ·007307 (!) Why? What *can* be the advantage of the double change, and how can it be defended? Notice the hopelessly mechanical rule given: "Alter multiplier till there is *only one figure in front of the decimal point*," and so on, *re* moving figures to "right" in multiplicand. Surely fundamental principles are best served by an avoidance of such expressions.

ON DIVISION OF DECIMALS BY A DECIMAL.

But where such a plan seems to me to produce fewer complications (*i.e.* in division of decimals), the plan as far as I can make out is abandoned and the divisor is made such a number as to contain one "singles" figure.

Thus, divide 12·45 by ·83.

Why is *this* plan not *always* convenient?

$$\frac{12{\cdot}45}{{\cdot}83}=\frac{1245}{83}=15. \quad \textit{Ans.}$$

(This method Mr. Borchardt (p. 75) quotes as "convenient in *some* cases.")

Here again, however, the rule as to "moving figures to right and left," etc. must surely be avoided for the expression: "Multiply numerator and denominator by the same power of 10." By this one gets a further application of the rule that "when numerator and denominator are multiplied or divided by the same numbers, the value of the fraction remains unaltered."

The use of the fractional form conduces to clearness of working and a neat use of factors; thus

$$40{\cdot}96 \div {\cdot}032$$

$$=\frac{40{\cdot}96}{{\cdot}032}=\frac{40960}{32}$$

$$=\frac{5120}{4}=1280. \quad \textit{Ans.}$$

[*N.B.*—Where I have used the word "singles" figure, I mean the so-called "units" figure, which is better avoided in this sense, as anything may be a unit; *e.g. tons*, in the case of the weight of a battleship; *millions of miles*, in astronomical measurement; *feet*, in the length of a desk, etc., etc.]

This fractional form is also of great help when one is asked to *deduce* one answer from one already obtained; thus

(*a*) Divide 430 by ·025, and (*b*) hence deduce result of 4·3 ÷ 2·5.

$$(a) \quad \left[\text{R.T. } 430 \div \frac{1}{40} = 430 \times 40 = 17200\right].$$

$$\frac{430}{{\cdot}025}=\frac{430000}{25}=\frac{86000}{5}=17200.$$

$$(b) \quad \frac{4{\cdot}3}{2{\cdot}5}=\frac{43}{25}=1{\cdot}72.$$

(Because 43 contains 25 *once*; then we come to decimal part, which, by deduction, must be ·72.)

I am aware that Arithmetic Committee's Report on "standard form" was unanimous. But I am extremely anxious to know whether such "standard form" is to be followed as a dogmatic order.

Aysgarth School. A. S. GRANT.

MODELS OF FUNCTIONS.

In a letter having the title "Classification and Mathematics," which appeared in *Nature* of May 28th, it was suggested that models of spaces and correspondences might usefully be employed in elementary mathematics. The present paper is restricted to the consideration of some simple one-to-one correspondences.

It will be taken for granted that, in order to arrive at any functional dependence or one-to-one correspondence of things, a classification and cross-classification is necessary. For example, the correspondence of English and French words tabulated in an English-French dictionary might be given as an illustration of a function (cf. Oliver Lodge, *Easy Mathematics*, 1905, p. 164). Here the words are divided into two classes, English and French, and are crossclassified by meaning.

In spite of the fact that boys make collections of all manner of things, the meaning of "classification and crossclassification" is usually not understood. It can, however, easily be illustrated by a model; for example, a number of pieces of cardboard, which can be classified by one respect as colour, and crossclassified by another as shape.

Therefore a one-to-one correspondence can be illustrated by a model. For example, if things differing in size can be divided into two classes by colour, and can be crossclassified by shape, then the crossclassification gives a one-to-one correspondence of sizes. Or, again, if things differing in colour can be divided into two classes by size, and can be crossclassified by shape, then the crossclassification gives a one-to-one correspondence of colours.

Persons who may consider it unlikely that works on the lines of Whitehead's *Axioms of Projective Geometry* will ever obtain a footing in schools—on account of their abstract character—and who find such a statement as that "geometry is the science of crossclassification" (p. 5) rather mysterious, will feel themselves on surer ground when dealing with models.

As further examples of one-to-one correspondence may be given :

(1) The correspondence of amounts of, say, corn and iron, which are of the same value. Here the crossclassification is by value.

(2) The correspondence of states of two changes given by the fact that they occur simultaneously. For example, the correspondence of a boy's weight to his height at the same time, or of weights hung on a wire to the extensions of the wire produced by the weights. As a general rule, the crossclassification in the case of the correspondences studied in chemistry and physics is given in this way.

(3) The correspondence of a thing to another from which it has been derived by effecting some particular change. For example, the correspondence of a liquid to its solid form. In symbolical work the crossclassification is so commonly given in this way that it is usual to call things which correspond operands, and the correspondence an operator. For want of another term, the word operand may be retained for general use, and a set of operands the correspondence of which to another set is under consideration will be called a "quantity." In this sense an English-French dictionary tabulates the correspondence of one quantity of words to another quantity of words. Here the operands are words, and in the case of correspondence of amounts of corn and iron the operands are amounts of corn and iron.

(4) The correspondence of persons indicated by saying that one is the father, grandfather, or husband, etc., of another. Since for the present we limit ourselves to one-to-one correspondences, it is necessary to suppose that to a father corresponds but one child, and to a husband but one wife. The correspondence of single operands, or of sets of operands to each other, is excluded. And therefore this limitation also requires the exclusion of the functions two, three, four, etc., which are not one-to-one correspondences.

(5) The correspondence of the subject of a transitive verb, such as the word "hit," to the object.

(6) The correspondence of a point in a district to a point in a map of the district, or of a point on an object to a point of its image in a photographic camera.

The nature of the properties used in the classification and crossclassification, or the reasons for the classification and crossclassification are not points which concern the mathematician, but what concerns him is the type of correspondence arrived at. Still less is it necessary to suppose that he is especially concerned with the correspondences of numbers. Indeed, to begin mathematics with the correspondences of numbers, for example the correspondence of a product of numbers to its factors, is not unlike beginning geometry with spaces whose elements are lines instead of points. It is beginning with the correspondences of correspondences instead of with correspondences themselves. And the correspondences which play the part of operands are, as already said, not even one-to-one correspondences.

As pointed out in the letter already quoted, it would seem more natural to consider the correspondence of operands to operands, and of operands to functions before going on to the correspondence of functions to functions.

The correspondences considered here are one-to-one correspondences, so that any operand occurs only once in its quantity. It is not supposed that the operands of a quantity have any particular order, nor has any limitation been put upon their number or nature.

If symbols a and b have been assigned to two quantities, then the fact of their having the correspondence f is expressed by the equation $a = fb$.

If symbols have been assigned to the operands of each quantity, then the function can be tabulated by a double column of these symbols, each symbol of one quantity being placed opposite the corresponding symbol of the other quantity.

If one quantity corresponds to another, the correspondence of the second to the first is said to be the inverse of this "direct" correspondence. For example, a French-English dictionary tabulates the inverse correspondence of that tabulated in an English-French dictionary. The function "son" is the inverse of father, grandson of grandfather, wife of husband.

The passive voice of a transitive verb is the inverse of the active voice. If the correspondence is got by effecting a change, then its inverse is got by undoing the change or reversing the operation. The distinction of direct and inverse is purely arbitrary, unless it happens that the correspondence is connected with one for which the terms have already been assigned.

Among the correspondences here considered, next to the distinction of direct and inverse, perhaps the most important division is that into functions where the same operands occur in both quantities, and functions where they are different. The function father is an example of the first kind, and the correspondence tabulated in an English-French dictionary, or the function husband, of the second kind. A model of a function of the first kind can be made by taking a colour-shape classification and crossclassification of sizes, the same sizes appearing in each quantity.

As another example of the first kind may be taken the correspondence of two points got by starting from one and going a given distance in a given direction, and so arriving at the other. Any point may be taken as commencement or termination, that is to say, any operand occurs in both quantities. Correspondences of this particular kind are called vectors. In Clark Maxwell's *Matter and Motion* a vector is defined as the operation by which a line is drawn, that of carrying a tracing point in a certain direction for a certain distance. This definition amounts to the same thing.

As already said, it is not the mode of arriving at a correspondence but the type of correspondence arrived at which mathematicians mainly consider.

Taking, then, functions of the first kind, the simplest type is evidently

that where each operand in one quantity corresponds to the same operand in the other. This correspondence is called one, and its direct and inverse are the same. Next to the function one, it is natural to take that where each of two corresponding operands is the same function of the other. This type is called a transposition, and here again the direct and inverse are the same. Then we come to the case of three operands, where one corresponds to another, this other to a third, and the third corresponds to the first. This type of correspondence may be called a treposition, and here the direct and inverse are not the same. Transpositions and trepositions, like other types of correspondence, can be illustrated by models or by a double column of symbols. Many other correspondences of this kind, consisting of perhaps several trepositions, or a transposition and a treposition, and so on, are considered in books on the Theory of Groups.

In the letter already referred to it was suggested that the words multiplication and addition, and terms such as factor ought to have the same meaning for all correspondences, and that it should agree with their meaning for the case of numbers.

That this is not at present the case is sufficiently shown by the large number of ways in which the product of two vectors is understood. If the word was used in the same way as for numbers, the term product of two vectors would be given to what is generally called their sum.

The meaning of product of two correspondences was defined in the letter. A multiplication or addition table of functions is an example of one-to-two correspondence of functions, and is therefore outside the range of this paper. But we may consider the question of choice of an intermediate quantity when a number of quantities have a one-to-one correspondence, and the meaning of "power" and "root" for some of the correspondences already referred to.

For example, in the case of a correspondence of words in two languages, this may be factorized into the product of the correspondence of one language to Esperanto and of Esperanto to the other. In the case of correspondence by value, the intermediate quantity in most countries consists of amounts of gold. In the case of correspondence of simultaneous states of changes, the intermediate quantity adopted is called the time.

A function must be of the kind having the same operands in both quantities in order that it may be multiplied by itself. Thus we can consider the square or cube of a function, such as "father," or of a vector, but not of the correspondence tabulated in an English-French dictionary. The square of "father" is grandfather, its cube is great-grandfather. If the crossclassification has been given by performing an operation, the square of the correspondence is given by a repetition of the operation. The square of a transposition is one, and the cube of a treposition is one. It may be noticed that if "root" is taken to mean the inverse of "power," then it is somewhat misleading to employ it also to mean a solution of an equation. By a cube root of one ought strictly to be meant a treposition, and not such an expression as $-\frac{1+\sqrt{-3}}{2}$.

C. Elliott.

Oundle.

QUADRATURE-FORMULAE IN RELATION TO MOMENTS.

Writers on mensuration do not seem to call attention to the convenience of quadrature-formulae for determining positions of centres of gravity, magnitudes of moments of inertia, etc.

Let u be the ordinate, for abscissa x, of a plane figure. Then the first moment of the figure with regard to $x=0$ is $\int uxdx$. If u is a rational integral algebraical function of x, then so also is ux; and the methods of quadrature apply to $\int uxdx$ as well as to $\int udx$. The principle can be extended to higher moments, and also to cases in which u is the area of the section of a solid by a plane whose abscissa is x.

In most cases what we really require is not the pth moment $\int ux^p dx$, but the mean value of x^p, i.e. $\int ux^p dx \Big/ \int udx$. It is then convenient to use the same quadrature-formula for $\int udx$ as for $\int ux^p dx$. Suppose that u is of degree m in x, and that $\lambda_0 v_0 + \lambda_1 v_1 + \ldots + \lambda_n v_n$ is an expression which gives the value of $\int vdx$ when v is of degree not greater than $m+p$ in x. Then $\int ux^p dx \Big/ \int udx = \Sigma\lambda ux^p \Big/ \Sigma\lambda u$, and we have only to take account of the mutual ratios of the λ's or the u's, not of their absolute values.

Suppose, for example, that we require the centre of gravity of a hemisphere of radius a. Here ux (x being measured along the axis) is of the third degree in x, and therefore Simpson's (the "prismoidal") formula applies. Taking the plane surface of the hemisphere to be $x=0$, so that the u's in the formula are the areas of the sections by $x=0$, $x=\frac{1}{2}a$, $x=a$, we have

λ (proportional to)	1	4	1
u (proportional to)	$2.2=4$	$3.1=3$	$4.0=0$
x	0	$\frac{1}{2}a$	a

and therefore, for the centre of gravity,

$$\bar{x} = \frac{1.4.0+4.3.\frac{1}{2}a+1.0.a}{1.4+4.3+1.0} = \frac{6a}{16} = \frac{3}{8}a.$$

Again, suppose we require the moment of inertia of a triangular lamina with regard to one side. In this case, also, ux is of the third degree in x. Measuring x from the particular side, and taking the distance of the opposite angle to be h, we have

λ	1	4	1
u	2	1	0
x^2	0	$\frac{1}{4}h^2$	h^2

so that
$$\text{mean value of } x^2 = \frac{4.\frac{1}{4}h^2}{2+4} = \frac{1}{6}h^2.$$

As a third example, let us find the moment of inertia of an ellipsoid with regard to a principal plane. Here ux is of the fourth degree, and Weddle's formula is convenient. The coefficients in the complete formula are

1, 5, 1, 6, 1, 5, 1 ;

but, since everything is symmetrical about the principal plane, we need only consider half the figure. We have then, if the axis is $2a$,

λ	3	1	5	1
u	3.3	4.2	5.1	6.0
x^2	0	$\frac{1}{9}a^2$	$\frac{4}{9}a^2$	a^2

and
$$\text{mean value of } x^2 = \frac{8+100}{27+8+25} \cdot \frac{1}{9}a^2 = \frac{1}{5}a^2.$$

W. F. SHEPPARD.

MATHEMATICAL NOTES.

275. [R. 6.] *Change of Kinetic Energy due to Mutual Action of two Particles, and loss of Kinetic Energy by Collision.*

If an action and reaction of any kind take place between two particles, the changes of momentum may be written :

$$I = m(u_1 - u_2) = m'(v_2 - v_1),$$

where m, m' are the masses, u_1, v_1 velocities before and u_2, v_2 velocities after the action.

Let V_1, V_2 be the relative velocities before and after, so that $V_1 = u_1 - v_1$, $V_2 = u_2 - v_2$.

Then $$I(m+m') = mm'(v_2 - v_1 + u_1 - u_2) = mm'(V_1 - V_2).$$

$$\text{Loss of Kinetic Energy} = \frac{1}{2}m(u_1^2 - u_2^2) + \frac{1}{2}m'(v_1^2 - v_2^2)$$

$$= \frac{1}{2}I\{(u_1 + u_2) - (v_1 + v_2)\}$$

$$= \frac{1}{2}I(V_1 + V_2)$$

$$= \frac{1}{2}\,\frac{mm'}{m+m'}\,(V_1^2 - V_2^2).$$

Hence, in general, there is loss of K.E. in all cases where the relative velocity is diminished, and a gain in cases (*e.g.* attraction) where the relative velocity is increased.

In cases of impact, where Newton's coefficient of restitution applies,

$$V_2 = -eV_1,$$

and loss of K.E. is $$\frac{1}{2}\cdot\frac{mm'}{m+m'}\cdot V_1^2(1-e^2),$$

which is positive since $0 < e < 1$.

Univ. Coll., Reading. A. L. Bowley.

[This method was certainly familiar at Cambridge in 1889, and was given by Mr. R. R. Webb to his pupils. It was, I believe, due to Professor Greenhill, who has drawn attention to it by examination questions at the University of London and elsewhere.

The physical meaning of I is the blow or impulse Pt between the colliding bodies, t being the time the collision lasts, and P the average pressure.]

C. S. J.

276. [A. 1. b.] *On certain Algebraical Factors.*

Let there be given a quadratic equation $ax^2 + bx + c = 0$ $(P=0)$, in which a, b, c are known coefficients. Then we can write

$$(\alpha) \ldots\ldots\ldots\ldots ax^2 = -(bx+c) \ldots\ldots\ldots\ldots (\alpha'),$$
$$(\beta) \ldots\ldots\ldots\ldots bx = -(ax^2+c) \ldots\ldots\ldots\ldots (\beta'),$$
$$(\gamma) \ldots\ldots\ldots\ldots c = -x(ax+b) \ldots\ldots\ldots\ldots (\gamma'),$$

and any arbitrarily manipulated equation such as $\phi(\alpha, \beta', \gamma) = \phi(\alpha', \beta, \gamma')$ is satisfied by the values of x given by $P=0$. Consequently

$$\phi(\alpha, \beta', \gamma) - \phi(\alpha', \beta, \gamma')$$

contains P as an algebraic factor.

For instance,
$$ax(ax^2+c)-b(bx+c)\equiv P.(ax-b),$$
$$bx^2(ax+b)-c(ax^2+c)\equiv P.(bx-c),$$
$$ax^3(ax+b)-c(bx+c)\equiv P.(ax^2-c),$$
$$a(ax^2+c)^2+b^2(bx+c)\equiv P.\{a^2x^2-abx+ac+b^2\},$$
and so on.

The same principle applies to equations of higher degrees.

Thus
$$(x^3+q)^3-p^3(px+q) \text{ is divisible by } x^3+px+q, \quad\text{(i)}$$
$$(px^2+q)^2(x+p)+qx^4 \quad ,, \quad ,, \quad x^3+px^2+q, \quad\text{(ii)}$$
$$(x^m+q)^2+p^2x^{2n-m}(px+q) \quad ,, \quad ,, \quad x^m+px^n+q), \quad\text{(iii)}$$
the quotients being
$$\text{(i) } x^6-px^4+2qx^3+p^2x^2-pqx+q^2-p^3,$$
$$\text{(ii) } p^2x^2+qx+pq,$$
$$\text{(iii) } x^m-px^n+p^2x^{2n-m}+q.$$
R. F. Davis.

277. [M^4. c. a.]. *On the equation of a certain spiral.*

Some years ago a Portuguese writer published an article on a certain "new" spiral to which he gave the name of "the binomial spiral of the first degree," defined by the equation
$$r=(a-a_0)\frac{\theta}{\pi}+r_0=a\theta+b. \quad\text{(1)}$$
With Messrs. Brocard, V. Jamet, and the late Prof. Longchamps, I hold that this equation may be reduced to the form $r=a\theta$, under which it is at once recognised as the spiral of Archimedes.

At first sight the curve (1) appears to be a conchoid of the curve $r=a\theta$, and in that case it is likely to be a different curve, for in most cases the conchoid of a curve is a curve of a higher degree. But in this instance the conchoid does not differ essentially from the original since (1) reduces to the form
$$r=a(\theta+k), \quad\text{(2)}$$
where k is a constant. It is, in fact, the spiral of Archimedes turned through a constant angle. Rodolphe Guimarães, Capt. R.E.

Elves, Portugal.

278. [V. a.]. *Notation of Binomial Coefficients.*

It is desirable that there should be a uniform and consistent notation for the binomial coefficients, and also for other expressions which occur in the binomial theorem, Vandermonde's theorem, Taylor's theorem, the ordinary interpolation formula, etc.

The most important expression is the binomial coefficient $\dfrac{n(n-1)\ldots\{r\}}{1.2\ldots\{r\}}$, where $\{m\}$ denotes the presence of m factors. In English works this is usually represented by a C, with n and r associated; *e.g.*, by C_r^n. A fundamental objection to this notation is that C_r^n properly represents, not the number given above, but the number of combinations of n things r together; it is true that the two are equal, but to use them as identical involves a confusion of thought. A further objection is that the n in C_r^n suggests the index of a power. This last difficulty is avoided by using ${}^nC_r$ or ${}_nC_r$; but these again are open to the objection that they take up more space, and further that the n, in printed matter, is liable to be attributed to something immediately preceding.

(The latter is one of the typographical faults of Chrystal's *Algebra*.)

Before choosing a symbol, we should see what are the various analogous expressions for which symbols are required. We have first the binomial theorem

$$(x+y)(x+y)\ldots\{n\}=x\,.\,x\ldots\{n\}+\ldots$$
$$+\frac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}\,.\,x\,.\,x\ldots\{n-r\}\,.\,y\,.\,y\ldots\{r\}+\ldots(1).$$

If, however, we compare this with Taylor's theorem

$$f(x+y)=f(x)+\frac{y}{1}\,.\,D_x\,.\,f(x)+\ldots+\frac{y\,.\,y\ldots\{r\}}{1\,.\,2\ldots\{r\}}\,.\,D_x\,.\,D_x\ldots\{r\}\,.\,f(x)+\ldots(2),$$

it will be seen that the $n(n-1)\ldots\{r\}$ really belongs to $x\,.\,x\ldots\{n-r\}$, and the $1\,.\,2\ldots\{r\}$ to $y\,.\,y\ldots\{r\}$.

We may in fact, when n is a positive integer, write the binomial theorem in the form

$$\frac{(x+y)(x+y)\ldots\{n\}}{1\,.\,2\ldots\{n\}}=\frac{x\,.\,x\ldots\{n\}}{1\,.\,2\ldots\{n\}}+\ldots+\frac{x\,.\,x\ldots\{n-r\}}{1\,.\,2\ldots\{n-r\}}\,.\,\frac{y\,.\,y\ldots\{r\}}{1\,.\,2\ldots\{r\}}+\ldots(3).$$

Again, we have Vandermonde's theorem

$$(x+y)(x+y-1)\ldots\{n\}=x(x-1)\ldots\{n\}+\ldots$$
$$+\frac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}\,.\,x(x-1)\ldots\{n-r\}\,.\,y(y-1)\ldots\{r\}+\ldots(4),$$

which is allied to the interpolation-formula

$$f(x+y)=f(x)+\frac{y}{1}\,.\,\Delta_x\,.\,f(x)+\ldots+\frac{y(y-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}\Delta_x\,.\,\Delta_x\ldots\{r\}\,.\,f(x)+\ldots(5),$$

and may more conveniently be written

$$\frac{(x+y)(x+y-1)\ldots\{n\}}{1\,.\,2\ldots\{n\}}=\frac{x(x-1)\ldots\{n\}}{1\,.\,2\ldots\{n\}}+\ldots+\frac{x(x-1)\ldots\{n-r\}}{1\,.\,2\ldots\{n-r\}}$$
$$.\,\frac{y(y-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}+\ldots(6).$$

Corresponding to (4) and (6), we have also

$$(x+y)(x+y+1)\ldots\{n\}=x(x+1)\ldots\{n\}+\ldots$$
$$+\frac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}\,.\,x(x+1)\ldots\{n-r\}\,.\,y(y+1)\ldots\{r\}+\ldots(7),$$

$$\frac{(x+y)(x+y+1)\ldots\{n\}}{1\,.\,2\ldots\{n\}}=\frac{x(x+1)\ldots\{n\}}{1\,.\,2\ldots\{n\}}+\ldots+\frac{x(x+1)\ldots\{n-r\}}{1\,.\,2\ldots\{n-r\}}$$
$$.\,\frac{y(y+1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}+\ldots(8).$$

Thus the expressions for which symbols are required are

(*a*) $x\,.\,x\ldots\{r\}$, (*b*) $n(n-1)\ldots\{r\}$, (*c*) $n(n+1)\ldots\{r\}$,

(*d*) $\dfrac{x\,.\,x\ldots\{r\}}{1\,.\,2\ldots\{r\}}$, (*e*) $\dfrac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}$, (*f*) $\dfrac{n(n+1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}$.

It is only for (*a*) that we have a settled notation, viz. x^r. For (*b*) $n^{(r)}$ is sometimes used; this gives parallel relations

$$D_x\,.\,x^r=r\,.\,x^{r-1},\qquad \Delta_n\,.\,n^{(r)}=r\,.\,n^{(r-1)}.$$

For (*e*) there are various notations, in addition to those mentioned above. H. Weber uses $B_r^{(n)}$. Other German writers sometimes use $\binom{n}{r}$ or $(n)_r$; in the second of these the brackets seem unnecessary. H. B. Fine uses n_r; but C. Smith uses n_r to denote (*b*), which, as seen above, may conveniently have an affix above the line. On the other hand, Sir A. G. Greenhill uses x_r to denote (*d*)

It certainly seems best that (*b*) and (*c*), like (*a*), should have affixes above the line, and that (*d*), (*e*), and (*f*), should have affixes below the line. The most suggestive notation would be

$$(a)\ \overset{(r)}{x},\qquad (b)\ {}^{(r)}n,\qquad (c)\ n^{(r)},$$
$$(d)\ \underset{(r)}{x},\qquad (e)\ {}_{(r)}n,\qquad (f)\ n_{(r)},$$

but this of course is impossible, and (*b*) and (*e*) would be open to the objection mentioned in the second paragraph above.

On the whole, since (*e*) is, after (*a*), the most important, I would suggest alternative symbols for it, with corresponding alternatives for (*f*). The following seem best :

$$(a)\ x^r \equiv x\,.\,x \ldots \{r\},\quad (b)\ n^{(r)} \equiv n(n-1)\ldots\{r\},\quad (c)\ n^{[r]} = n(n+1)\ldots\{r\},$$

$$(d)\ x_r \equiv \frac{x\,.\,x\ldots\{r\}}{1\,.\,2\ldots\{r\}},\quad (e)\ n_{(r)} \equiv \binom{n}{r} \equiv \frac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}},\quad (f)\ n_{[r]} \equiv \begin{bmatrix} n \\ r \end{bmatrix}$$
$$\equiv \frac{n(n+1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}.$$

Thus the formulae (1)–(6) above become

$$(1)\ (x+y)^n = x^n + \binom{n}{1}x^{n-1}y^1 + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots,$$

$$(2)\ f(x+y) = f(x) + y_1\,.\,D_x f(x) + \ldots + y_r\,.\,D_x{}^r f(x) + \ldots,$$

$$(3)\ (x+y)_n = x_n + x_{n-1}y_1 + \ldots + x_{n-r}y_r + \ldots,$$

$$(4)\ (x+y)^{(n)} = x^{(n)} + \binom{n}{1}x^{(n-1)}y^{(1)} + \ldots + \binom{n}{r}x^{(n-r)}y^{(r)} + \ldots,$$

$$(5)\ f(x+y) = f(x) + \binom{y}{1}\Delta_x f(x) + \ldots + \binom{y}{r}\Delta_x{}^r f(x) + \ldots,$$

$$(6)\ (x+y)_{(n)} = x_{(n)} + x_{(n-1)}y_{(1)} + \ldots + x_{(n-r)}y_{(r)} + \ldots,$$

or

$$\binom{x+y}{n} = \binom{x}{n} + \binom{x}{n-1}\binom{y}{1} + \ldots + \binom{x}{n-r}\binom{y}{r} + \ldots,$$

with corresponding formulae in place of (7) and (8). W. F. SHEPPARD.

279. [K. 21.]. *Two approximate geometrical constructions for inscribing a Nonagon in a circle.*

1. Col. Weldon's Construction :

Let $\alpha\beta\gamma$ be the inscribed circle, centre I, of the equilateral triangle ABC cutting IA, IB, IC at α, β, γ respectively. With centre β and radius $\beta\gamma$ describe a circle cutting the circle whose centre is A and radius AB, at X. Then BX is a side of the nonagon.

2. Mr. J. Houghton Spencer's Construction :

Let PQ, QR be two sides of a hexagon escribed to a circle, touching the circle at B and C. Bisect QC at a. Join Pa cutting the circle at X. Then BX is a side of the nonagon.

Error. The angle $BAX = 40^\circ\ 5'\ 57{\cdot}6''$ showing an error of approximately $6'$ equivalent to $\frac{1}{30}$ part of an inch on an arc of 20 inch radius.

The remarkable part of the two constructions given above, and discovered independently, is that they are in reality identical as is seen from the following geometrical proof.

Let QR be one side of an escribed hexagon touching $\odot$ at C.

Bisect QC at a. Join Pa.

It is required to prove that Pa passes through X, the point where the arc centre β and radius $\beta\gamma$ cuts the circle BXC, Y being the other point of intersection of these two circles.

Produce Pa to cut AC at N.

Produce $A\beta$ to cut Pa at M.

XY is the radical axis of the two circles $Y\gamma X$, BXC and $\therefore$ passes through a, since aC, aQ are tangents to these two ⊙ˢ and equal.

If $AB=2d$, then $\beta E=\frac{2d}{\sqrt{3}}$, $AE=d$. Hence $\frac{\beta E}{EA}=\frac{2}{\sqrt{3}}$.

$$\left.\begin{aligned} CQ \text{ and } AP \text{ are parallel and } Ca=\tfrac{1}{2}AP&=\frac{d}{\sqrt{3}} \\ \text{and } CN=\tfrac{1}{2}AN=\tfrac{1}{3}AC&=\frac{2d}{3} \end{aligned}\right\};$$

$$\therefore \frac{aC}{CN}=\frac{\sqrt{3}}{2}=\frac{EA}{E\beta}.$$

Hence angle βAE = angle CaN = angle QaP.

Hence angle QPM = angle BAM.

Hence $BPAM$ are concyclic.

$\therefore$ Angle PMA = angle PBA = a right angle.

$\therefore$ $A\beta$ is perpendicular to Pa.

But $A\beta$ (line of centres) is perpendicular to radical axis YXa,

$\therefore$ YXa and Pa coincide,

i.e., Pa passes through X. S. de J. Lenfestey.

280. [R. 6. a. β.] *On the Kinetic Measure of a Force.*

That the force on a particle is proportional to the time-rate of change of its momentum is, of course, an article of faith; but that the same statement may be applied to a body without any reservation as to what is meant by this term appears to be open to discussion.

To take an example: Suppose a railway train, moving uniformly, to enter a tunnel; the part within the tunnel at any instant is clearly a "body" and is acquiring momentum so long as the train is only partly within the tunnel. But we do not say that this fact points to the existence of unbalanced force on this portion of the train, for the change of momentum arises here from a flux of mass into the tunnel, and our "body" does not consist from time to time of the same particles.

Let μ be the mass per unit length of the train, v its velocity. In the time dt the momentum brought into the tunnel is clearly $\mu v dt\times v$. We must then subtract μv^2 from the rate at which momentum is accumulating in the "body" to arrive at the true kinetic measure of the force upon it. Similarly, if we fix attention on the part left within the tunnel as the train is emerging we must add μv^2 to the rate of change of its momentum in order to form a just estimate of the resultant force upon it; which force should be zero in both cases, as the motion is uniform. (Cf. Lamb's *Motion of Fluids*, 1st ed., § 12.)

The problem of snow-sliding down a roof, alluded to by Prof. G. H. Bryan in our May number, is, in my opinion, incorrectly solved in one of our text-books. No friction or adhesion are allowed for at all, and yet, wonderful to say, the acceleration of the portion left on the roof is only $\frac{1}{3}$ that of a freely-sliding particle!

This extraordinary result is obtained thus: Let b be the breadth of this portion, x its length, μ the mass of snow per unit area, a the slope of the roof.

$$-\frac{d}{dt}(\mu b x\dot{x}) = \mu b x g \sin a\,; \quad \text{......(1)}$$

$$\therefore\ \frac{d}{dt}(x\dot{x}) = -xg \sin a,$$

$$x\dot{x}\frac{d}{dx}(x\dot{x}) = -x^2 g \sin a,$$

$$\tfrac{1}{2}(x\dot{x})^2 = -\tfrac{1}{3}x^3 g \sin a,$$

$$\tfrac{1}{2}\dot{x}^2 = -\tfrac{1}{3}xg \sin a,$$

$$\ddot{x} = -\tfrac{1}{3}g \sin a,$$

whereas the equation of motion should, I think, have been

$$-\mu b x\ddot{x} = \mu b x g \sin a, \quad \text{......(2)}$$

whence $$\ddot{x} = -g \sin a.$$

It will be seen that the term $\mu b\dot{x}^2$, representing the rate of efflux of momentum across the spouting, must be added to the first member of (1) to make the equations (1) and (2) agree, as the tunnel theory would indicate. That (2) gives a result less in accordance with observation than (1) may easily be accounted for by the fact that the friction as compared with the weight of the snow is by no means negligible. (Discussion invited.)

C. E. M'Vicker.

281. [D. c.]. *Note on Taylor's Theorem.*

Bertrand's proof of Taylor's Theorem with Lagrange's remainder is given in Williamson's *Differential Calculus*; other text-books give very similar proofs. I have been in the habit of using the following modification of Bertrand's proof for over 25 years, but thought it too small a thing to print. Now I see an appeal to print any *little thing* in the *Gazette*; and I further notice that Prof. Love gives two series in expanding e^x, which are only particular cases of my general series.

With the usual limitations as to continuity, which need not be repeated here, we have to find R if

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{\lfloor 2}f''(x) + \ldots + \frac{h^{n-1}}{\lfloor n-1}f^{(n-1)}(x) + \frac{h^n}{\lfloor n}R.$$

Now form a new function of z, from the right-hand side of the above, by writing $x+z$ for x and $h-z$ for h; or let

$$\phi(z) = f(x+z) + (h-z)f'(x+z)$$
$$+\frac{(h-z)^2}{\lfloor 2}f''(x+z) + \ldots + \frac{(h-z)^{n-1}}{\lfloor n-1}f^{(n-1)}(x+z) + \frac{(h-z)^n}{\lfloor n}R.$$

Then $\phi(z)$ takes the value $f(x+h)$ when $z=0$ and when $z=h$.

Hence $\dfrac{d\phi(z)}{dz}$ must vanish for some value of z between 0 and h, say when $z=\theta h$.

Now $$\frac{d\phi(z)}{dz} = \frac{(h-z)^{n-1}}{\lfloor n-1}\{f^{(n)}(x+z) - R\}\,;$$

$$\therefore\ f^{(n)}(x+z) - R \text{ vanishes when } z=\theta h, \text{ or } R = f^{(n)}(x+\theta h).$$

When I first used this method I included the term $f(x+h)$ in $\phi(z)$, giving either this term or all the others the minus sign. The introduction of z

of course does not affect $f(x+h)$. But after some years it struck me that the inclusion of $f(x+h)$ in $\phi(z)$ was unnecessary.

Still, if we take

$$\phi(z)=f(x+h)-f(x+z)-(h-z)f'(x+z)$$
$$-\frac{(h-z)^2}{\lfloor 2}f''(x+z)-\dots$$
$$-\frac{(h-z)^{n-1}}{\lfloor n-1}f^{n-1}(x+z)-\frac{(h-z)^n}{\lfloor n}R,$$

the proof goes on in the same manner.

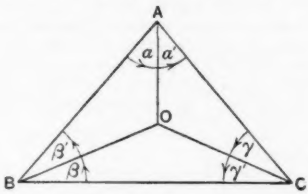
If $f(x)$ is ϵ^x, and if in the second form of $\phi(z)$ we put 0, x, 1 for x, z and h respectively, $\phi(z)$ becomes the $\phi(x)$ of Prof. Love's series (1) in number 66 of the *Gazette*; but if we put 0, x, b for x, z and h, then $\phi(z)$ becomes $\psi(x)$ of Prof. Love's later series. PERCY J. HARDING.

282. [K. 20. e.] *Trigonometrical Note.*

If O, A, B, C are four points in a plane, and we denote the six angles BAO, OAC, CBO, OBA, ACO, OCB by α, α', β, β', γ, γ', then

$$\alpha+\alpha'+\beta+\beta'+\gamma+\gamma'=\pi,$$
$$\sin\alpha\sin\beta\sin\gamma=\sin\alpha'\sin\beta'\sin\gamma'.$$

Hence if any four of the six are given the rest can be found.



There are two cases. The two angles to be found may be one dashed and the other undashed. Or both may be dashed or both undashed.

In the first case we know the ratio of the sines and the sum of the angles. These may, therefore, be found by the process for solving a triangle when two sides and the included angle are given, so that the solution is unique and real.

In the second case we know the product of the sines and the sum of the angles. These may, therefore, be found from

$$\cos(x-y)=\sin x\sin y+\cos(x+y),$$

so that the solution is two-fold and not necessarily real.

Under the first case we have two standard problems, the determination of the distance between two visible but inaccessible objects, and Pothenot's problem (Todhunter and Hogg, § 242).

Under the second case we have the following problem :—the bearings of two objects whose relative position is known are observed, required the points in a given line through the point of observation at which the join of the objects subtends a given angle. E. J. NANSON.

283. [A. 1. b.] *Note on the Power Inequality.*

If x, y have the same sign and p, q are rational numbers, then

$$\frac{x^p-y^p}{x^q-y^q} \text{ lies between } \frac{p}{q}x^{p-q} \text{ and } \frac{p}{q}y^{p-q}. \quad\dots\dots(1)$$

Proof. If n is a positive integer,

$$\frac{x^n-y^n}{x-y}=x^{n-1}+x^{n-2}y+\dots+y^{n-1}.$$

Hence, if x, y have the same sign,

$$\frac{x^n-y^n}{x-y} \text{ lies between } nx^{n-1} \text{ and } ny^{n-1}.$$

It readily follows that (*Gazette*, p. 321),

$$\frac{x^n-y^n}{x^{n-1}-y^{n-1}} \text{ lies between } \frac{n}{n-1}y \text{ and } \frac{n}{n-1}x.$$

Hence by multiplication it follows that (1) is true when p, q are any positive integers.

It then readily follows from (1) that

$$\frac{x^p-y^p}{x^{p-q}-y^{p-q}} \text{ lies between } \frac{p}{p-q}y^q \text{ and } \frac{p}{p-q}x^q. \quad\dots\dots\dots\dots(2)$$

From this it follows that (1) is true when one of the integers p, q is negative.

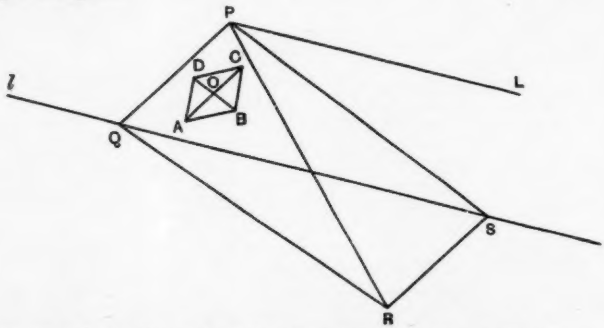
Using (2) again it then follows that (1) is true when the integers p, q are both negative.

The extension of (1) to fractional values of p, q follows at once by writing $x=X^m$, $y=Y^m$, where m is the L.C.M. of the denominators of p, q.

E. J. NANSON.

284. [K. 21. a. α.] *Note on the problem: "Given a parallelogram, construct a parallel to a given line through a given point."*

This problem is a well-known one, and solutions of it based on principles of homology are given in Cremona's *Projective Geometry* and Mr. J. W. Russell's *Treatise on Elementary Geometry*. The construction can however be effected as an immediate deduction from the ruler construction for the fourth harmonic to three given lines. Thus let l be the given line, P the point, and $ABCD$ the parallelogram.



Let AC, BD, the diagonals of the given parallelogram meet in O.

Draw PQ the fourth harmonic to PA, PO, PC, meeting l in Q.

Then since $AO=OC$, PQ is parallel to AC.

Draw PS the fourth harmonic to PD, PO, PB, meeting l in S.

Then since $DO=OB$, PS is parallel to DB.

Similarly, draw QR the fourth harmonic to QD, QO, QB, meeting SR the fourth harmonic to SC, SO, SA, in R.

Then $PQRS$ is a parallelogram.

The required line is PL the fourth harmonic to PQ, PR, PS, for since PR bisects QS, PL is parallel to QS, *i.e.* to l.

M. I. TRACHTENBERG.

REVIEWS.

An Introduction to the Theory of Groups of Finite Order. By H. HILTON. (Clarendon Press.) 1908.

Group-theory, and in particular the theory of groups of finite order, is one of the most recent developments of mathematical thought. During the last twenty-five years, and especially in the latter half of them, a very large number of memoirs and several books have been published dealing with various branches of the subject and making material advances both in the general theory and in matters of detail. Far the greater part of these investigations have been due to Continental and American mathematicians; indeed it cannot be denied that hitherto group-theory has failed to attract much serious attention in England. No course of lectures, specifically devoted to the theory of groups of finite order, has ever been delivered in an English University; while in most American and in many Continental Universities such courses occur regularly. Why the subject should have failed here to excite the interest that attaches to it elsewhere it is difficult to say; though a probable surmise is that the fact must depend in some way on the nature of the training of junior students. It is not, of course, suggested that any of the developments of group-theory are suitable for introduction into the course for junior students. But while this is certainly so, it is equally certain that the attention of the student might be more often directed to the nature of the processes and operations which he carries out, and not so entirely concentrated, as it generally is, on their result.

A definite case in point is the process of inversion at a circle in questions of plane geometry. Many a student, at a comparatively early stage, learns to use this process with some skill in dealing with particular geometrical problems. But how many of them recognize that, quite apart from the particular result, the process of inversion at a circle is a definite geometrical operation which replaces every point by another definite point, and which, when repeated, leaves every point unchanged? By laying more stress on this point of view (in dealing with inversion, reciprocation, projective transformation, etc.) the student would, at all events in the region of geometry, soon become familiar with the idea of definite operations, which can be carried out in succession in any order, the result being always another definite operation; *i.e.* he would have acquired in a particular class of cases the necessary ideas which underlie all group-theory.

In the fourth chapter of his *Introduction to the Theory of Groups of Finite Order* Mr. Hilton gives an admirable account, in the main geometrical, of the operations which lead to the simpler point-transformations in space of two and of three dimensions. The two chapters which precede it are devoted to elementary properties of the operations of permutation and of linear substitution as applied to a finite set of symbols. Having thus paved the way, by familiarizing the reader in a number of particular cases with the idea of definite operations which can be combined together, the author enters in the fifth chapter on the theory of groups of finite order. This chapter is of necessity in large part devoted to the explanation of terms and ideas which will be new to the reader, such as group, sub-group, conjugate set, isomorphism, factor-group; and it will not be found easy reading by the beginner. Each section of the text is here, as elsewhere, followed by a number of illustrative questions proposed to the reader, while in the notes at the end of the book the method of treating the more difficult of these questions is indicated. It is doubtful whether, in this chapter at least, it would not have been wiser to give in the text itself illustrations of some of the new ideas introduced.

The general principles of Chapter V. are applied in the next three chapters to permutation-groups, groups of linear substitutions and groups of movements. The author has wisely avoided going into too great detail in these applications. Thus in the chapter on permutation-groups it is shown that every group of finite order can be represented as a regular permutation-group; the various methods of representing a group as a non-regular permutation group are indicated; the ideas of transitivity and primitivity are explained; and the chapter ends with a proof of the important theorem that the alternating group is, except in the case of four symbols, simple. On the other hand, the chapter contains none of those

enumerations "of all possible types" which are of very doubtful utility and quite undoubted wearisomeness to the reader.

In Chapter VII. the proof that every finite group of linear substitutions has a positive Hermitian invariant form really involves a forward reference to Chapter XV.; for of the two sets of variables in the form, one is transformed by the operations of the given group and the other by those of the "inverse" group. This chapter also contains an explanation of the idea of "irreducibility" in connection with a group of linear substitutions, and a proof of the fundamental theorem in this connection.

In Chapter IX. the principal properties of Abelian groups are discussed with a clearness and simplicity of method which leaves nothing to be desired. An account of groups whose orders are powers of primes is given in Chapter XI. It is perhaps a pity that this chapter could not have followed Chapter XIII., in which the series of "derived" and of "adjoined" groups are considered; for these series have certainly their most important application at present to prime-power groups.

Sylow's theorem is the subject of Chapter XII., and the isomorphisms of a group with itself, or automorphisms, form the subject of Chapter X. In the last chapter of the book a short account is given of the theory of group-characteristics, the theory being actually worked out for the case of Abelian groups, but merely stated for the general case.

Reference has already been made to the large number of examples which are given throughout the book. They are extremely well chosen: being real illustrations, and not conundrums of examination type. They undoubtedly form a most important part of the book, and in connection with them the author's advice, given in the preface, must be borne in mind. He writes: "Students who have no previous acquaintance with the subject should work a few of these examples after reading each section." This is the more important since in the text itself there is little or no illustrative matter.

Mr. Hilton is to be congratulated on the success with which he has accomplished the task he set himself. His book is a real introduction; and at the same time carries the reader sufficiently far in all branches of the subject to enable him to continue his studies on any lines he may wish. The interest in the latter chapters is necessarily centred on the abstract theory; but the chapters on permutations, those on linear substitutions and especially those on kinematics, should appeal to many to whom the abstract theory may be almost repulsive. The appearance of this book ought to do a good deal to diminish the apathy with which its subject is treated in this country.

W. BURNSIDE.

An Introduction to the Theory of Infinite Series. By T. J. I'A. BROMWICH. xiv+511 pp. London: Macmillan & Co. 1908.

Notwithstanding the great importance of the theory of infinite series in all departments of analysis, the English reader has hitherto been very poorly provided with reliable sources of information. The chapter devoted to the subject in text-books of algebra and trigonometry usually contain a good many obscure and misleading if not positively incorrect statements, while even the most scholarly treatises of this class, such as Dr. Hobson's *Trigonometry*, are prevented by considerations of space from giving more than the elements of the theory. The best account hitherto available in English is probably that contained in the *Introduction to the Theory of Analytical Functions* by Professors Harkness and Morley (a book which the publishers have unfortunately allowed to go out of print), while there are valuable though necessarily somewhat brief discussions in Professor Carslaw's recent *Fourier's Series*, and embedded in Dr. Hobson's monumental treatise on *Functions of a Real Variable.*

Mr. Bromwich has therefore rendered an important service to mathematical students and teachers by writing an elaborate and scholarly treatise devoted explicitly to series and to allied portions of the theory of limits.

The subject is introduced by a chapter on sequences and limits. The four chapters on real numerical series which follow contain all the ordinary as well as many much more elaborate tests for the convergence of real single and double series, as well as a discussion of the characteristic properties of absolutely and of non-absolutely convergent series. Infinite products are then dealt with more

briefly, the correspondence with series being in most of the propositions so close as to render elaborate independent proofs unnecessary. Then follow three chapters on series the terms of which are functions of a variable, dealing first with the general theory of uniform convergence, and then with the application of the preceding theories to power series and to trigonometrical series.

The theory of real series and products being thus completed, a single long chapter introduces the necessary modifications and extensions, when the variables and the coefficients are complex.

So far the book has been concerned with theories of which the elements at least are familiar to English mathematicians. The final chapter breaks quite new ground. Though many mathematicians have doubtless been perplexed by the fact that the older analysts, notably Euler, frequently worked with divergent series but were seldom led thereby into errors, it has only been during the last quarter of a century that any serious attempt has been made to construct a scientific theory of divergent series, which is in effect a general calculus operating with divergent series as instruments and leading to results of demonstrable accuracy when interpreted according to fixed rules. Isolated results are to be found in Cauchy and in earlier papers of Stokes, but the effective foundation of the modern theory dates from papers published by Stieltjes and M. Poincaré in 1886. Although a good deal has been written on certain aspects of the subject by a few English mathematicians, there has hitherto been no connected account of it in English, and Mr. Bromwich's long and interesting chapter is a very valuable addition to our literature.

Mr. Bromwich ends his book with three appendices. The first contains an account of irrational numbers, based mainly on Dedekind; the second gives an interesting sketch of the theory of the exponential and logarithm, based on the definition of the latter as an integral; while the third, extending to more than fifty pages, deals with the convergence of integrals and allied questions.

Of the importance and value of Mr. Bromwich's book as a storehouse of accurate information on the subjects with which it deals there can hardly be two opinions. Its suitability as a text-book, to be really read by students, is more open to question. It is stated to be based on courses of lectures on elementary analysis given at Queen's College, Galway. Either the base must have been very slight for such a superstructure, or Irish students must have a capacity of assimilating analysis of which their Cambridge brethren may well be envious. No treatment of the subject on this scale could help being difficult, but the author does not appear to have given the comparatively elementary student as much help as he might have done, by emphasising the really important and fundamental theorems in the different parts of the subject. For example, the statement and proof of the familiar test of convergence, $\mathrm{Lt}\,(u_{n+1}/u_n) < 1$, though given in a rather more elaborate form, occupy only two lines, in the form of a corollary from Kummer's much more general test. Doubtless from the author's point of view Kummer's test is both more interesting and more important than the simple one quoted, but this is not the case for the ordinary student. Many readers will also resent the frequent *forward* references, especially to the elaborate appendix on Integrals. Frail human nature will in too many cases shirk the labour of looking up these references, though without them the text becomes obscure or inconclusive. The arrangement suggests that Mr. Bromwich originally intended to confine himself almost entirely to series and to use only the elements of the Infinitesimal Calculus, but that in the course of writing he became interested in the allied theories of definite integrals and added his Appendix III. rather as an afterthought.

On the other hand, individual sections are hardly ever obscure; a number of familiar propositions are established by interesting methods which, if not strictly original, are at any rate unfamiliar, and difficult proofs have been simplified without losing their rigour by the use of elementary differential and integral calculus. There are an enormous number of difficult examples and some easy ones.

I may add one criticism of a technical nature. Continental writers habitually divide series into two classes only, convergent and divergent. Mr. Bromwich, in accordance with the common practice of English text-books, further subdivides non-convergent series into oscillatory and divergent series, but gives so wide a definition of the former that he refuses the name divergent to such a series $1+x+x^2+\ldots$ when x is negative and numerically greater than unity. I believe this to be inconvenient even in the case of real series, but it is still more

troublesome in the complex case. As a matter of fact, when Mr. Bromwich discusses this point in connection with complex series, he gives an ambiguous definition, while his practice is at times certainly inconsistent with his earlier definitions. If the distinction between oscillatory and other non-convergent series is wanted at all, I believe that it would be better to use qualifying adjectives, as some German writers do, such as improperly divergent and properly divergent, and to restrict the former name to non-convergent series, such as $1-1+1-1\ldots$, in which the sum of any number of terms always lies between two fixed limits.

But I should be sorry to end this review on a note of criticism. I began by complaining of the poverty of English writing on series; let me end by expressing the opinion that Mr. Bromwich's book is decidedly more complete, and in other ways more satisfactory, than any of the continental treatises with which it is at all comparable. ARTHUR BERRY.

King's College, Cambridge.

Modern Geometry. by C. GODFREY, M.A., and A. W. SIDDONS, M.A. 162 pp. Cambridge University Press.

The authors state in their preface that the treatise covers the schedule of Modern Plane Geometry required for the Special Examination in Mathematics for the Ordinary B.A. degree at Cambridge, and that they have also had regard to the requirements of students in Physics and Engineering. The time seems almost to have arrived for entering a protest against mathematical text-books being framed too much to suit the special needs of this or any other favoured class of students, and it is a pity that a book bearing the comprehensive title *Modern Geometry* should be subject to such narrow limitations.

The work consists of 13 chapters bearing the headings: the sense of a line, infinity, the centroid, the triangle, the theorems of Ceva and Menelaus, harmonic section, pole and polar, similitude, miscellaneous properties of the circle, the radical axis and coaxal circles, inversion, orthogonal projection, cross-ratio, and the principle of duality. The first three chapters are brief, but clear, although we hardly think that a student's objection to the convention that "a point at infinity may be infinitely distant from itself" would be satisfied by the explanation that "points at infinity do not enjoy all the properties of ordinary points."

In the chapter on the triangle, the authors do not hesitate to employ trigonometrical notation, and they give us a large and well selected number of the properties which exist between the elements of a triangle and the radii of the associated circles, but although they deal pretty fully with the N.P.C. they omit to give us a demonstration of Feuerbach's Theorem. For the past 30 years this has been an object of investigation to many geometricians with more or less success, and the recent proof by Ramaswami Ayar, as revised by R. F. Davis, is as near the ideal proof as can be expected, and is of a sufficiently elementary character to be admitted into a chapter dealing with the properties of the N.P.C. In chapter vii the authors do not seem quite happy in their treatment of pole and polar, owing to their exercising the self-denial of making no reference to imaginary points, due no doubt to their respect for the feelings of the practical engineer. We would suggest that they would have done better to have taken as their fundamental proposition Ex. 262, viz. "if H be the harmonic conjugate of a fixed point T with regard to the points in which a line through T cuts a fixed circle, the locus of H is a straight line," and then taken T and the locus of H as their definition of pole and polar. The various properties would then have followed as the natural consequences of harmonic section.

In the chapter giving miscellaneous properties of the circle we have four sections dealing with orthogonal circles, the circle of Apollonius, Ptolemy's Theorem, and contact problems. In §§ 1, 4 the proofs are, as in many other cases, left to the reader. This withholding of the proof, when judiciously exercised, is a distinct advantage, but in the case of describing a circle touching three given circles it would have been advisable to give references to some of the numerous text-books which have treated this classical problem fully.

The chapter on coaxal circles is carefully done and well illustrated. Here the student's attention might have been drawn to the existence of the second common chord, so as to prepare him to find these lines occurring in pairs in conics.

The chapters on inversion and orthogonal projection are full of interest, the proofs and figures of the former being especially neat. The chapter on cross-ratio is somewhat disappointing. Of course in a subject which occupies the greater part of Chasles's *Géométrie Supérieure* it is difficult to give very much in a dozen pages, but considering its fascinating character and its supreme importance we expected, at least, an introduction to homographic ranges and their double points when they are on the same straight line, and the fundamental properties of involution, but we are not even given a method of finding the fourth point of a range in which the other three points and the cross-ratio are known.

The work concludes with a very suggestive chapter on the principle of duality, and deals with the complete quadrilateral and quadrangle, self-polar triangle, and Desargues' Theorem on triangles in perspective, followed by a useful index. In addition to the text there are 679 exercises which appear to be well chosen, and will afford the reader abundant scope for his ingenuity. But we note that Nos. 617, 650, and 654 are respectively the same as 601, 608, and 606.

Taken as a whole we can give a hearty welcome to the book, which is well arranged with good figures, and we especially welcome the few notes of human interest which we should like to see more generally introduced into mathematical text-books. They might with advantage be expanded in a second edition, for which we anticipate an early demand, *e.g.* no student can help being interested when he is told that Pascal's Theorem, as far as relates to a pair of lines, was known to Euclid (B.C. 300), and employed by him without proof in his books on Porisms, and that a proof was given by Pappus 600 years afterwards. In 1640 the theorem was given by Pascal without proof as a property of the circle, and 166 years later its correlative was published by Briauchon, in 1806. An enquiring pupil, however, would hardly be satisfied with the note on p. 20 that Apollonius (260-200 B.C.) studied and probably lectured at Alexandria, and was nicknamed ϵ.

J. MILNE.

Magic Squares and Cubes. By W. S. ANDREWS and others. Chicago: The Open Publishing Company.

The construction of magic squares has a curious fascination which appeals to the mystic and the ignorant, as well as to the mathematician. The theory of the formation of simple squares has long been worked out, save for two problems. One of these, probably beyond our powers, is the number of squares of the fifth (or any higher) order. The other is a rule to enable us from any given square of the nth order to produce another square of order $n+2$ by adding $2(n+1)$ to each number in the original square, and surrounding it by a border of the remaining $4(n+1)$ numbers. It is not difficult to border a given square empirically, but a definite rule to enable us to do it in general terms is still wanting. Of late attention has been mainly directed to the construction of squares where additional conditions are imposed, and recently French mathematicians have turned their attention to the formation of double and triple magic squares.

In the earlier chapters of the book mentioned above, the condition is imposed that the square must be such that the sum of any two numbers geometrically equidistant from its centre shall be constant. A large number of the usual constructions are not affected by this restriction. The subject is treated mathematically, and bordered squares are discussed, but the writer does not seem to be aware that the essential part of his diagonal rule for odd squares was given by De Laloubère in the seventeenth century, and that the construction of compound magic squares was mentioned by Montucla in his edition of Ozanam's *Recreations*.

The interest of the book lies, however, rather in its philosophical and quasi-paradoxical parts. Dr. Carus chats about this aspect of the subject, and gives examples of magic squares in China and India. In the Introduction he says: "Magic squares are a visible instance of the intrinsic harmony of the laws of number, and we are thrilled with joy at beholding this evidence which reflects the glorious symmetry of the cosmic order." And again, they "are like a magic mirror which reflects a ray of the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos not less than the mysterious depths of the human mind." This will appeal to the mystic philosopher more than to the prosaic mathematician.

Mr. Browne also contributes to the work an attempt to explain the Platonic

Numbers (*Republic*, ix. 587-8, viii. 546) by a composite magic square of the order 27. He says frankly that this explanation is conjectural, but it is plausible, and may be commended to Platonic students.

Trinity College, Cambridge. W. W. ROUSE BALL.

Les Nombres positifs. Exposé des théories modernes de l'arithmétique élémentaire. Par M. STUYVAERT. Gand [no date; preface dated June 22nd, 1906]. Pp. xii + 133.

M. Stuyvaert's doctrine is that arithmetic borrows a few fundamental concepts (like 'number' and 'addition') and axioms (like the commutativity of addition) from experience, and builds on these its direct operations. The inverse operations implied by these direct ones are not always possible, and "give rise to symbols, such as negative numbers, etc. ; and the mechanism of the calculus of these symbols is pure convention, subject only to the condition of not contradicting the preceding theories, in the case where these symbols represent possible operations. As for the sciences of application, they are subject to this calculus by means of conventions also, and postulates particular to each of them." There is nothing in the exposition which would show that it was not written thirty years ago, and consequently, although it is more carefully done than the part devoted to principles in some modern text-books, it can hardly be called *modern*,—neglecting as it does the facts, now fairly well-known, of the definability of all arithmetical concepts, the provability of all arithmetical axioms, the independence of arithmetic of experience, and the untenability of the view that the 'numbers,' in the extended sense of arithmetic, are ever mere symbols. PHILIP E. B. JOURDAIN.

Five Figure Mathematical Tables. By A. DU PRÉ DENNING. (Longmans, Green & Co. 2s.)

These are intended for school and laboratory work. By the use of a large page (11″ × 7″) and a small but clear type the author contrives to bring the difficult tables included in his collection (Logarithms, Antilogarithms, Trigonometrical Functions, Squares, Cubes, etc.) into a small number of pages (21). Thus Circular Measures, Sines, Tangents and their Logarithms, are all comprised within 6 pages. In the Tables of Logarithms and Antilogarithms the weak parts of both are avoided. Thus the Logs. of the numbers from 4·0000 to 9·9999 are given, and, *on the opposite page*, the Antilogs. for those from 1·0000 to 4·0644, all the differences being consequently small. Useful tables of constants and formulae are added. It seems a timely publication and can be recommended for Laboratory use in cases where 5-fig. work is necessary. We should have been glad to see the superfluous 10 rejected from the Logarithmic Trigonometrical function. Its retention throws unnecessary difficulties in the way of the computer.

E. M. LANGLEY, M.A.

Problems on Strength of Materials. By W. K. SHEPARD, Ph.D., Instructor in Yale University. (Ginn & Co.) 1907. pp. 70. Price 6s.

This book contains 568 examples in applied mechanics, dealing with shearing force, bending moments, the compression of struts, and simple cases of torsion and of stresses in cylinders, pipes, and plates exposed to fluid pressure.

The examples are nearly all numerical, but answers are not given "so as to emphasize that the goal is a proper solution." The policy of this omission, in a book containing, very properly, several cases differing only in numerical data, appears doubtful.

There is a short but pithy note on the design of riveted joints at p. 12, and the book concludes with a table of constants. C. S. J.

Integration by Trigonometric and Imaginary Substitution. By C. O. GUNTHER, with an Introduction by J. BURKITT WEBB, Professors at the Stevens Institute of Technology. (Constable & Co.) 1907. Pp. 78. Price 5s. net.

The object of this book is to assist the student of applied mathematics to effect certain integrations (mostly of quadratic surds) by the free use of imaginary and trigonometrical substitutions.

That a book on pure mathematics should be written by one whose chief interest

is in practical mathematics is well—an engineer can appreciate the intellectual state of an engineer student.

He rightly upholds the importance of directness and naturalness of method. Many students regard the suggestion of putting $\sqrt{a^2+x^2}=z-x$ in order to integrate $\frac{1}{\sqrt{a^2+x^2}}$ as an outrage and a fraud, and so it is if put forward as a natural way of working out an unknown integral. We have the fullest sympathy with the author's desire to systematize methods of integration, and with every wish to acknowledge that an argument which would make the editors of the *Quarterly Journal* (let us say) stare and gasp, may convince a student in whom the more closely reasoned logical proof would merely produce suspicion, confusion, and dislike. But where is the student who finds comfort in the following extract from Professor Webb's introduction?

p. 2—"If then -1 be recognised as a quantity, $\sqrt{-1}$ must also be a quantity, for it is inconceivable that the quantitative nature of -1 can be destroyed by the operation of finding its square root when this operation has no such effect on other quantities."

This is a fair specimen of the value of the introduction.

Professor Gunther's share in the book is accurately written, but his choice of substitutions is not always judicious. Thus at p. 59 he integrates $\frac{1}{(a^2-x^2)\sqrt{b^2-x^2}}$ by putting $x=a\sin\theta$ when $x=b\sin\theta$ is more natural and also more effective.

C. S. JACKSON.

Cinq études de géométrie analytique. By DR. M. STUYVAERT, Répétiteur à l'Université de Gand. (Librairie scientifique E. van Goethem, Rue des Foulons, 1, Gand.)

The sub-title is "Applications diverses de le théorie des matrices et de l'élimination," and this gives a fair idea of the contents of the work. If the finding of the equation of a surface locus in space comes to the elimination of a parameter t between two equations, the expression of the condition that the equations in t should have more than one root common leads to the equations of singular lines on the surfaces. This condition, and consequently the equations of the singular lines, are expressed in the form of a matrix, whose elements are polynomial functions of the co-ordinates. The author investigates some of the properties of curves and other varieties in space of three or more dimensions whose equations can thus be expressed by the vanishing of a matrix; but he does not go far, and the work does not appear to present any great novelty. F. S. M.

Il Passato ed il Presente delle principali Teorie Geometriche. By GINO LORIA. 1907. (Carlo Clausen Hans Rinck Succ., Torino.)

The name of the Professor of Higher Geometry at the University of Genoa ranks deservedly high among those of the erudite of European reputation, and a third edition of his volume on the past and present of geometrical theories will not be welcomed the less because he has added a sketch of the developments of the last ten years. It is fortunate that those to whom the book will be of use will be no more than irritated by the appalling frequency of misprints in the titles of papers and memoirs, for otherwise the value of the work would be very considerably impaired. Misprints have been detected after a cursory perusal on pp. 25, n., 30, 37, 47, 48, 75, 85, 93, n., 94, 105, 114, 121, 129, 145, and n., 161, 171, n., 185, 192, 193, 195, 200, 201, 230, 233, 234, 235, 289, 324, 332, 355, 357, 358, 365, 393, 394, 425, 427, 429, 431, 440, 445, 447. "Poincarré" appears again and again, but is spelled correctly in the index. *En revanche*, "Russel" appears in both text and index, as does "Richemond." W. H. and F. S. Macaulay are confounded. The "tree cusped hypocycloid" reminds us of a similar delightful misprint we once noticed in the *Bulletin de Mathématiques Spéciales*, now defunct, "*hypocycloide à trois remboursements*." We suppose it is inevitable that in dealing with large masses of papers written in practically all European tongues that there should be some misprints, but there lurks in one's mind the suspicion that careful investigation will here discover a multitude. The sketch of recent developments extends to 130 pages. W. J. G.

The Schoolmasters' Year Book and Directory. 1908. Pp. 460+590. 7s. 6d. net. **The Public Schools' and Preparatory Schools' Year Book.** 3s. 6d. net. Pp. 684. **The Girls' School Year Book (Public Schools).** Pp. 576. 2s. 6d. net. 1908. (Messrs. Swan, Sonnenschein.)

These three volumes are now so well known to teachers and to the public in general that there is little need for us to add to the paean of praise to which the compilers are entitled for the success of their labours and which they appear to have received from the Press. In the first of the books named above we looked with pardonable pride for the Mathematical Association among the Educational Associations. We found *en passant* the Polyglot Club, the Navy League, the League of Empire and the London Mathematical Society, each of which is in a more or less restricted sense "educational." But we could not find the Mathematical Association. The *Gazette*, on the other hand, receives full recognition. We have tested all the volumes in many ways, and with the single exception to which we have alluded, they have been equal to the tests. W. J. G.

QUERIES.

(59) Solutions invited:

1. Triangles are described about an ellipse (foci S, H), all having the same circumcircle (centre O). $SOHQ$ is a ▱^{m}. Prove that the locus of the orthocentres of these $\triangle^{s}$ is a circle with centre Q.

2. If OS be produced to meet the circumcircle in L, prove that LH will meet that locus circle on the line SQ. Hence it follows that $OS.OH=$ rectangle contained by radii of the 2 $\odot^{s}$.
See Note 217 on page 406 of *Math. Gazette*, No. 60.

3. Draw $SM \parallel LH$ to meet SH in M, and prove $LM=$ major axis of ellipse.

E. P. Rouse.

(60) Let A be a point on any circle and A' a diameter of any conic.

Let P be another point on the same circle and P' another diameter of the same conic; then, if the angle subtended at the circumference of the circle by the arc AP equals the angle between A' and P', a one-to-one correspondence is established between P and P'.

Any chord which has for its extremities points of the circle corresponding to conjugate diameters of the conic is called a conjugate chord.

Prove: (1) That all conjugate chords are concurrent.
(2) That if PQ be a conjugate chord and O the point of concurrence, the square of the semi-diameter P' is inversely proportional to $PO.PQ$.
(3) That certain properties of diameters in a conic can thus be presented as properties of a circle.

R. S. Ball.

ANSWER TO QUERY.

[**43, p. 212.**] M. Brocard quotes, from P. Tannery's edition of Fermat, a letter from Wallis, dated $\left.\begin{matrix}\text{21st Nov.}\\ \text{1st Dec.}\end{matrix}\right\}$1657, in which he writes, "Hence the moments, the ratio of which is compounded of those of the masses, and their distances from the centre of the balance . . ." The first use of the word is usually attributed to Varignon, who was born only three years before the letter in question was written. Mr. H. Braid reminds the readers of the *Intermédiaire* that momentum is derived from *moveo*, and really signifies movement, especially that of the scales of a balance. So that any mathematician writing in Latin would naturally use the word *momentum.* M. Paul Tannery recalls the fact that the Greeks gave the name ῥοπή to the tendency of the scales to fall, whence also their word

ἰσορροπία for equilibrium. This tendency as proportional to the weight and the arm was given in the so-called *Mechanics* of Aristotle as well as in Eutocius' Commentary on Archimedes' *Equilibrium of Planes*, though Archimedes does not betray that he has any cognisance of the relation. Commandin's translation of Eutocius, 1506, translates the Greek word by momentum, evidently the proper technical expression. The development of the idea of moment, outside the case of parallel forces, is due to Galileo. He first extended it to the theory of simple machines (in his *Mechanics*, written about 1593, and published for the first time, translated into French by Mersenne, in 1634). He afterwards generalised it fully, at the same time being the first to introduce a principle equivalent to that of virtual velocities. This was in his *Discorso intorno alle cose che stanno in sù l'acqua*, etc.

EDITOR.

ERRATA.

5th line up, p. 304. *For* $b_{\frac{3}{32}}$, *read* $b^{\frac{3}{32}}$.
8th line, p. 306. *For* ·314, *read* ·243.

BOOKS, ETC., RECEIVED.

The Annals of Mathematics. Second series. Vol. IX. No. 4. Edited by ORMOND STONE and others. July, 1908. 2s. (Longmans, Green.)

On the Spherical Representation of a Surface. PAUL SAUREL. *The Absolute Minimum in the Problem of the Surface of Revolution of Minimum Area.* Miss M. E. SINCLAIR. *Note on the Roots of Bessel Functions.* C. N. MOORE. *A Smooth Closed Curve composed of rectilineal Segments with Vertex Points which are nowhere dense.* E. R. HEDRICK. *Evaluation of the Probability Integral.* F. GILMAN. *On a Second Theorem of the Mean.* C. N. HASKINS. *Another Proof of the a Theorem in Multiplying Perfect Numbers.* R. D. CARMICHAEL. *A Theorem concerning equal Ratios.* J. L. COOLIDGE. *Note on certain iterated and multiple Integrals.* W. A. HURWITZ.

Invariants of Quadratics Differential Forms. E. WRIGHT. Pp. 90. 2s. 6d. net. (Cambridge University Press.)

The Elementary Theory of the Symmetrical Optical Instrument. J. G. LEATHEM. Pp. 74 2s. 6d. net. 1908. (Cambridge University Press.)

Gazeta Matematica. Jan.-July. 7 lei per ann. Edited by I. IONESCU. (Nicolaevici, Bucharest.)

Annaes Scientificos da Academia Polytechnica do Porto. Edited by GOMES TEXEIRA. Nos. i. and ii. Vol. III. (University Press, Coimbra.)

Formulario Mathematico. Edited by G. PEANO. 5th edition. Vol. V. of the complete edition. Pp. 275-463. Fasc. 2. 1908. (Bocca, Turin.) *Formulario Mathematico. Praefatione.* Table of Symbols and Bibliography. Pp. xxxvi.

Teacher's Handbook to Blackie's Adaptable Arithmetics. Book II. Pp. viii, 102. 1s. 1908. Book V. Pp. viii, 106. 1s. (Blackie.)

Blackie's Adaptable Arithmetics. Book II. *Compound Rules.* Book V. *Commercial Rules.* 4d. each. Pp. 82, 76. 1908. (Blackie.)

Blackie's Elementary Modern Algebra. By R. C. BRIDGETT. Pp. 192, 40. With Answers, 1s. 6d. 1908. (Blackie.)

Magic Squares and Cubes. By W. S. ANDREWS. Pp. vi, 199. 1908. (Open Court, Chicago.)

Differential Calculus for Beginners. By A. LODGE, M.A. 3rd edition. Pp. xviii, 300. 4s. 6d. 1908. (Bell.)

Elementary Mensuration. By W. M. BAKER and A. A. BOURNE. Pp. 144. 1s. 6d. 1908. (Bell.)

Introductory Mechanics. By E. J. BEDFORD. Pp. x, 140. 1s. 6d. 1908. (Longmans, Green.)

American Journal of Mathematics. Edited by F. MORLEY. Vol. XXI. No. 3. July 1908. $1·50.

The Determination of the Conjugate Points for Discontinuous Solutions in the Calculus of Variations. O. BOLZA. *Mathematical Logic as based on the Theory of Types.* B. RUSSELL. *Invariantive Reduction of Quadratic Forms in the G.F.* [2^n]. L. E. DICKSON. *The Motion of a Particle attracted towards a fixed Centre by a Force varying inversely as the fifth Power of the Distance.* W. D. MACMILLAN.

A New Algebra. By S. BARNARD and J. M. CHILD. Vol. I. containing Parts I.-III. Pp. vii, 371. 2s. 6d. 1908. (Macmillan.)

The Eton Algebra. Part I. by P. SCOONES and L. TODD. Pp. xxv, 184. 2s. 6d. 1908. (Macmillan.)

[A compilation of examples for Beginners. No Bookwork. Specimen examples to secure uniformity of method.]

Elementary Mensuration. By W. M. BAKER and A. A. BOURNE. Pp. 144. 1s. 6d. 1908. (Macmillan.)

Histoire des Mathématiques. By W. W. ROUSE BALL. Translated (with additions from Chasles, Biot, Bertrand, Mach, Duhem, Darboux, and the translator) by Lieut. L. Freund. Vols. I., II. Pp. 422, 270. 1906-7. (Hermann.)

Les Origines de la Statique. By P. DUHEM. Vols. I. and II. Pp. 360, 364. 1905-6. 20 frcs. (Hermann.)

Die Lehre von den Geometrischen Verwandtschaften. By R. STURM. Vol. II. Pp. viii, 346. 1908. 16 m. (Teubner.)

Wahrscheinlichkeitsrechnung. By E. CZUBER. Vol. I. *Fehlerausgleichung-Kollectivmasslehre.* Pp. x, 410. 1908. 12 m. (Teubner.)

Vorlesungen über Differentialgeometrie. By R. v. LILIENTHAL. Pp. vi, 368. 1908. 12 m. (Teubner.)

Das Prinzip der Erhaltung der Energie. 2nd Edition. By MAX PLANK. Pp. xvi, 278. 1908. 6 m. (Teubner.)

Analytische Geometrie der Ebene. By C. RUNGE. Pp. 198. 1908. 6 m. (Teubner.)

Vorlesungen über bestimmte Integrale und die Fourierschen Reihen. By J. THOMAE. Pp. vi, 182. 1908. 7.80 m. (Teubner.)

The Physics of Earthquake Phenomena. By C. G. KNOTT. Pp. xii, 280. 14s. net. (Clarendon Press.)

Experimental Elasticity. A Manual for the Laboratory. By G. F. C. SEARLE, F.R.S. Pp. xvi, 187. 5s. net. 1908. (Cam. Univ. Press.)

Algebra for Secondary Schools. By C. DAVISON, Sc.D. Pp. viii, 623. 6s. 1908. (Cam. Univ. Press.)

The Principles of Mechanics. H. CREW. Pp. v, 295. 1908. 6s. net. (Longmans, Green.)

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The Analytical Geometry of the Conic Sections. By E. H. ASKWITH. Pp. xiv, 443. 1908. (A. & C. Black.)

Elliptische Funktionen. By K. BOEHM. Vol. I. *Theorie der elliptischen Funktionen aus analytisches Ausdrücken entwickelt.* Pp. xii, 356. 1908. 8.60 m. 1908. Sammlung-Schubert, XXX. (Göschen, Leipzig.)

Gruppen und Substitutionentheorie. By E. NETTA. Pp. viii, 175. 5.20 m. Sammlung-Schubert, LV. (Göschen, Leipzig.)

Coordinate Geometry. By J. H. GRACE and M. ROSENBERG. Pp. viii, 346. 4s. 6d. 1907. (Univ. Tut. Press.)

Elementary Algebra. By P. ROSS. Part. I. Pp. xii, 484; 65. 4s. 6d. Part II. Pp. viii, 273-484; 45-64. 2s. 6d. 1908. (Longmans.)

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